

# Exotic baryons and monopole excitations in a chiral soliton model

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**Abstract.** We compute the spectra of exotic pentaquarks and monopole excitations of the low-lying  $\frac{1}{2}^+$  and  $\frac{3}{2}^+$  baryons in a chiral soliton model. Once the low-lying baryon properties are fit, the other states are predicted without any more adjustable parameters. This approach naturally leads to a scenario in which the mass spectrum of the next to lowest-lying  $J^\pi = \frac{1}{2}^+$  states is fairly well approximated by the ideal mixing pattern of the  $\mathbf{8} \oplus \overline{\mathbf{10}}$  representation of flavor  $SU(3)$ . We compare our results to predictions obtained in other pictures for pentaquarks and speculate about the spin-parity assignment for  $\Xi(1690)$  and  $\Xi(1950)$ .

**PACS.** 12.39.Dc Skyrmions – 14.20.-c Baryons (including antiparticles) – 14.80.-j Other particles (including hypothetical)

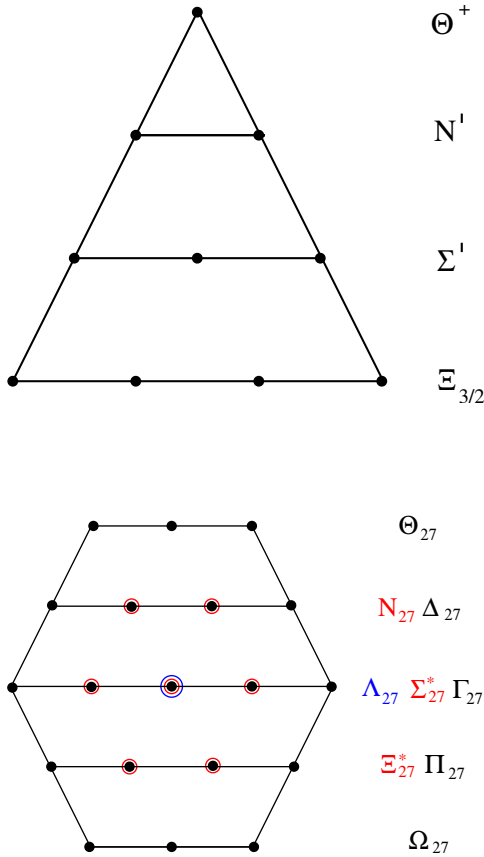
## 1 Introduction

Although chiral soliton model predictions for the mass of the lightest exotic pentaquark, the  $\Theta^+$  with zero isospin and unit strangeness, have been around for about two decades [1], the study of such pentaquarks as potential baryon resonances became popular only very recently when experiments [2, 3] indicated their existence. These experiments were stimulated by a chiral soliton model estimate that suggested [4] that such exotic baryons might have a width<sup>1</sup> that is much smaller than those typical for hadronic decays of baryon resonances [4, 5]. Such narrow resonances could have escaped detection in earlier analyses. Then very quickly the experimental observations initiated exhaustive studies on the properties of pentaquark baryons. Comprehensive lists of such studies are, for example, collected in refs. [9, 10].

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<sup>1</sup> In chiral soliton models the direct extraction of the interaction Hamiltonian for hadronic decays of resonances still is an unresolved issue. Estimates are obtained from axial current matrix elements [4–7]. In view of what is known about the related  $\Delta \rightarrow \pi N$  transition matrix element [8], such estimates may be questioned. It is also worth noting that the computation of this particular decay width has a long and notable history. The interested reader may trace it from ref. [5].

Chiral soliton models are a common platform for such studies because higher-dimensional  $SU_F(3)$  irreducible representations that contain exotic pentaquarks emerge almost naturally. In these models states with baryon quantum numbers are generated from the soliton by canonically quantizing the collective coordinates that parameterize the large-amplitude fluctuations associated with (would-be) zero modes. When extending the model to  $SU_F(3)$  the lowest states are members of the flavor octet and decuplet representations for  $J^\pi = \frac{1}{2}^+$  and  $J^\pi = \frac{3}{2}^+$ , respectively. Upon inclusion of flavor symmetry breaking the physical states acquire admixtures from higher-dimensional representations. For the  $J^\pi = \frac{1}{2}^+$  baryons those admixtures originate dominantly from the antidecuplet,  $\overline{\mathbf{10}}$ , and the **27**-plet [11–14]. The particle content of these representations is depicted in fig. 1. In addition to states with quantum numbers of octet baryons these representations obviously also contain states with quantum numbers that *cannot* be built as three-quark composites. These states contain (at least) one additional quark-antiquark pair. Hence the notion of exotic pentaquarks. So far, the  $\Theta^+$  and  $\Xi_{3/2}$  with masses of  $1537 \pm 10$  MeV [2] and  $1862 \pm 2$  MeV [3] have been observed, even though the single observation of  $\Xi_{3/2}$  is not undisputed [15] and awaits confirmation. In soliton models these two resonances are identified as members of the antidecuplet  $SU_F(3)$  representation. Therefore they



**Fig. 1.** A sketch of the  $SU_F(3)$  representations  $\overline{10}$  and  $27$  with their exotic-baryon members:  $\Theta^+$ ,  $\Xi_{3/2}$ ,  $\Theta_{27}$ ,  $\Lambda_{27}$ ,  $\Gamma_{27}$ ,  $\Pi_{27}$  and  $\Omega_{27}$ . As usual various isospin projections are plotted horizontally while states of different hypercharge are spread vertically.

should carry the quantum numbers  $I(J^\pi) = 0(\frac{1}{2}^+)$  for  $\Theta^+$  and  $\frac{3}{2}(\frac{1}{2}^+)$  for  $\Xi_{3/2}$ . We stress that these quantum numbers are not yet confirmed by experiment. The channels for which the resonance analyses were performed are  $nK^+$  and  $pK_S^0$  for  $\Theta^+$  [2] and  $\Xi^-\pi^-$  for  $\Xi_{3/2}$  [3]. Hence the strangeness quantum numbers of these exotics are without ambiguities:  $S(\Theta^+) = 1$  and  $S(\Xi_{3/2}) = -2$  and the isospin of  $\Xi_{3/2}$  should at least be  $\frac{3}{2}$ .

It is now suggestive to wonder about the nature of states in those higher-dimensional representation that do carry quantum numbers of three-quark composites. In the antidecuplet these are  $N'$  and  $\Sigma'$ , cf. fig. 1. Similarly,  $\Delta_{27}$ ,  $\Sigma_{27}$  and  $\Xi_{27}^*$  have the same quantum numbers as the ordinary  $\Delta$ ,  $\Sigma^*$  and  $\Xi^*$  from the decuplet, respectively. In ref. [4], for example, the  $N' \in \overline{10}$  was identified as the  $N(1710)$ -resonance to adjust the overall mass scale of the antidecuplet. Though that approach correctly predicted the mass of  $\Theta^+$ , the prediction for  $\Xi_{3/2}$  turned out to be incompatible with the observation. It was then quickly realized that the  $N' \in \overline{10}$  and  $\Sigma' \in \overline{10}$  could easily mix with nucleon and  $\Sigma$  states that are members of an additional octet (besides the ground-state octet). This pic-

ture leads to the description of pentaquarks as members of the direct sum  $\mathbf{8} \oplus \overline{\mathbf{10}}$  and could be motivated both in quark-diquark [16] and chiral soliton [17] approaches. However, the latter description did not provide any dynamical origin for the additional octet. Of course, it is very natural to view this additional octet as a monopole (or radial) excitation of the ground-state octet. The fact that such monopole excitations would mix with the corresponding states in the antidecuplet was recognized already some time ago [18, 19] and a dynamical model was developed not only to investigate such mixing effects but also to describe static properties of the low-lying  $J^\pi = \frac{1}{2}^+$  and  $J^\pi = \frac{3}{2}^+$  baryons. Technically that was accomplished by not only quantizing the collective coordinates that parameterize the flavor (and spatial) orientation of the soliton but also its spatial extension. This approach was further justified by the observation that the proper description of baryon magnetic moments requires a substantial feedback of flavor symmetry breaking on the soliton size [20].

The philosophy of chiral soliton models is to first construct a chiral Lagrangian for meson fields and determine as many model parameters as possible from meson properties. Baryons then emerge as solitons of this meson theory making the approach very predictive in the baryon sector. In the context of pentaquark studies, however, a commonly adopted (model-independent) strategy is to write down all operators in the space of collective coordinates up to a given order in  $1/N_C$  and/or flavor symmetry breaking (measured by the strange-quark mass) and determine the unknown coefficients from baryon properties [4, 10, 21–23]. Once the additional collective coordinate for the monopole excitation is included, the number of symmetry-allowed operators in the Hamiltonian is no longer limited to just a few. Therefore such model-independent approaches turn out not to be very predictive because not sufficient information is available to determine all coefficients. Rather a fully dynamical calculation must be performed in a given model. This has the additional advantage that no assumption is used as input about the nature of states in the higher-dimensional  $SU_F(3)$  representations. In particular, no such input is necessary to fit the mass scale for states in the  $\overline{10}$  and  $27$  representations.

In the studies of refs. [18, 19] the spectra for radial excitations of the low-lying  $\frac{1}{2}^+$  and  $\frac{3}{2}^+$  baryons were computed in the framework of an  $SU_F(3)$  chiral soliton model. In addition the version of the model discussed in ref. [19] was shown to describe the static properties of those baryons fairly well. Later the mass of the recently discovered  $\Theta^+$  pentaquark was predicted with reasonable accuracy in the same model [6, 24]. Here we will therefore employ exactly that model without any further modifications to predict masses of the exotic  $\Xi_{3/2}$  and of additional exotic baryons that originate from the  $SU_F(3)$  representations  $\overline{10}$  and  $27$ , respectively. The latter are exotic pentaquarks with  $J^\pi = \frac{3}{2}^+$  and may be considered as partners of  $\Theta^+$  and  $\Xi_{3/2}$  in the same way as the  $\Delta$  is the partner of the nucleon. We will also reanalyze the  $J^\pi = \frac{1}{2}^+$  excited baryons in view of the recently developed  $\mathbf{8} \oplus \overline{\mathbf{10}}$  scenario.

Essentially the model has only a single free parameter that has been fixed earlier [18,19] to provide an acceptably nice picture of baryon properties. In this way, it is also possible to discuss the relevance of mixing between rotational and vibrational modes when attempting to predict the spectrum of pentaquarks. This is crucial for pentaquarks because the mass differences to the nucleon of these modes are of the same order in the  $1/N_C$  expansion [25]. Hence mixing effects are potentially large even if they are mediated by the (small) flavor symmetry breaking.

The paper is organized as follows. In sect. 2 we will review the specific soliton model. As will be described in sect. 3 the quantization of both the rotational and the monopole degrees of freedom leads to the baryon states that mix with the (exotic) pentaquarks in the antidecuplet and the **27**-plet. The numerical results on the spectrum will be presented in sect. 4. We compare our results to those obtained in other descriptions for the pentaquark spectrum that are also formulated in the framework of mixing between excited octet and antidecuplet baryons in sect. 5. We summarize in sect. 6. A final word on notation: we will call exotics or exotic pentaquarks all baryons that must contain at least one antiquark to saturate the quantum numbers. On the other hand we call pentaquarks all baryonic members of  $SU_F(3)$  representations whose Young tableaux need one antiquark even though the quantum numbers of some of these baryons can be built from three-quark states; in this case they are non-exotic.

## 2 The soliton model

In this section we will describe the soliton model that we will use later to analyze monopole and rotational degrees of freedom. For the present investigation we will employ the Skyrme model extended by a scalar field as motivated by the trace anomaly of QCD. There are reasons to believe that extensions with vector mesons or chiral quarks are more realistic [12]. However, and as discussed in ref. [6], technical subtleties may occur when quantizing the radial degrees of freedom in more realistic soliton models like those with chiral quarks and/or vector mesons. We therefore consider the scalar meson extension of the Skyrmion as adequate.

To be specific we will follow the treatment of ref. [19] where the soliton model contains a scalar meson,  $\sigma$  and eight pseudoscalar mesons  $\phi^a$ ,  $a = 1, \dots, 8$ . The scalar meson field parameterizes  $H = \langle H \rangle \exp(4\sigma) = \frac{\beta(g)}{g} G_{\mu\nu}^a G^{a\mu\nu}$  which is introduced as an effective order parameter for the gluon field to imitate the dilation anomaly equation of QCD [26],  $-\partial_\mu D^\mu = H + \sum_i m_i \bar{\Psi}_i \Psi_i$ . The vacuum expectation value  $\langle H \rangle \sim (0.30-0.35 \text{ GeV})^4$  can be extracted from sum rule estimates for the gluon condensate [27]. Eventually the fluctuating field  $\sigma$  may be identified as a scalar glueball. The effective mesonic action reads

$$\Gamma = \int d^4x (\mathcal{L}_0 + \mathcal{L}_{\text{SB}}) + \Gamma_{\text{WZ}} . \quad (1)$$

The flavor-symmetric part involves both the chiral field<sup>2</sup>  $U = \exp(i\lambda^a \phi^a / f_a)$  as well as the scalar fluctuation  $\sigma$

$$\begin{aligned} \mathcal{L}_0 = & -\frac{f_\pi^2}{4} e^{2\sigma} \text{tr} (\alpha_\mu \alpha^\mu) + \frac{1}{32e^2} \text{tr} \left( [\alpha_\mu, \alpha_\nu]^2 \right) \\ & + \frac{1}{2} \Gamma_0^2 e^{2\sigma} \partial_\mu \sigma \partial^\mu \sigma \\ & + e^{4\sigma} \left\{ \frac{1}{4} [\langle H \rangle - 6(2\delta' + \delta'')] - \sigma \langle H \rangle \right\} , \end{aligned} \quad (2)$$

with  $\alpha_\mu = \partial_\mu U U^\dagger$ . Assuming the canonical dimensions  $d(U) = 0$  and  $d(H) = 4$  it is straightforward to verify that the Lagrangian (2) yields the dilation anomaly equation for  $m_i = 0$ . The terms which lift the degeneracy between the pseudoscalar mesons of different strangeness are comprised in the flavor symmetry breaking part of the Lagrangian,

$$\begin{aligned} \mathcal{L}_{\text{SB}} = & \text{tr} \left\{ \left( \beta' \hat{T} + \beta'' \hat{S} \right) e^{2\sigma} \partial_\mu U \partial^\mu U^\dagger U \right. \\ & \left. + \left( \delta' \hat{T} + \delta'' \hat{S} \right) e^{3\sigma} U + \text{h.c.} \right\} , \end{aligned} \quad (3)$$

where the flavor projectors  $\hat{T} = \text{diag}(1, 1, 0)$  and  $\hat{S} = \text{diag}(0, 0, 1)$  have been introduced. The coupling of the scalar field in  $\mathcal{L}_{\text{SB}}$  is such as to reproduce the explicit breaking in the dilation anomaly equation [28]. The various parameters in eqs. (2) and (3) are determined from the well-known masses and decay constants of the pseudoscalar mesons:

$$\begin{aligned} \beta' & \approx 26.4 \text{ MeV}^2, & \beta'' & \approx 985 \text{ MeV}^2, \\ \delta' & \approx 4.15 \times 10^{-5} \text{ GeV}^2, & \delta'' & \approx 1.55 \times 10^{-3} \text{ GeV}^4 . \end{aligned} \quad (4)$$

Then the only free parameters of the model are the Skyrme constant  $e$  and the glueball mass,

$$m_\sigma^2 = \frac{4\langle H \rangle + 6(2\delta' + \delta'')}{\Gamma_0^2} . \quad (5)$$

As in ref. [19] we will always take  $m_\sigma \approx 1.25 \text{ GeV}$ . This corresponds to  $\langle H \rangle = 0.01 \text{ GeV}^4$  and  $\Gamma_0 = 180 \text{ MeV}$ . Finally the scale invariant Wess-Zumino term is most conveniently presented by introducing the one-form  $dUU^\dagger = \alpha_\mu dx^\mu$ ,

$$\Gamma_{\text{WZ}} = \frac{iN_C}{240\pi^2} \int \text{tr} [(dUU^\dagger)^5] . \quad (6)$$

The above-described model possesses a static soliton solution of hedgehog structure

$$U_0(\mathbf{r}) = \exp[i\boldsymbol{\tau} \cdot \hat{\mathbf{r}} F(r)] \quad \text{and} \quad \sigma(\mathbf{r}) = \sigma_0(r) \quad (7)$$

which is characterized by the two radial functions  $F(r)$  and  $\sigma_0(r)$  [26,29]. This configuration has non-trivial topology and the corresponding winding number is identified with the baryon charge. The profile functions  $F(r)$  and  $\sigma_0(r)$  are determined from the Euler-Lagrange equations to the action (1) with boundary conditions suitable for baryon number one.

<sup>2</sup> Here the normalization coefficients  $f_a$  refer to the pseudoscalar decay constants.

### 3 Quantization of the soliton

Besides the baryon number the configuration (7) does not carry quantum numbers of physical baryon states such as spin or isospin. These are generated by canonical quantization of the collective coordinates which are introduced as time-dependent parameters to describe large-amplitude fluctuations around the soliton. Apparently these are the rotations in coordinate and flavor spaces. The hedgehog configuration (7) is embedded in the isospin space. Hence the coordinate rotations may be expressed as rotations in the flavor subspace of isospin and it suffices to consider flavor rotations. These rotational modes are parameterized by  $A(t) \in SU_F(3)$  and their quantization will thus lead to states that carry good spin and flavor quantum numbers. In addition the energy surface associated with scale or breathing transformations of the soliton is known to be shallow, at least in a large vicinity of the stationary point [30,31] allowing for large-amplitude fluctuations. This is even more the case when the dilatation symmetry is respected. For this reason it is suggestive also to introduce a collective coordinate,  $\mu(t)$  for the spatial extension of the soliton. The quantization of this mode will then describe the monopole excitations. We therefore consider the following time-dependent meson configuration that emerges from the soliton (7),

$$\begin{aligned} U(\mathbf{r}, t) &= A(t)U_0(\mu(t)\mathbf{r})A^\dagger(t) \quad \text{and} \\ \sigma(\mathbf{r}, t) &= \sigma_0(\mu(t)r) . \end{aligned} \quad (8)$$

We then substitute this parameterization into the action (1) and obtain the Lagrangian for the collective rotations  $A(t)$  and the monopole vibration  $x(t) = [\mu(t)]^{-3/2}$

$$\begin{aligned} L(x, \dot{x}, A, \dot{A}) &= \frac{4}{9} \left( a_1 + a_2 x^{-\frac{4}{3}} \right) \dot{x}^2 \\ &- \left( b_1 x^{\frac{2}{3}} + b_2 x^{-\frac{2}{3}} + b_3 x^2 \right) + \frac{1}{2} \left( \alpha_1 x^2 + \alpha_2 x^{\frac{2}{3}} \right) \sum_{a=1}^3 \Omega_a^2 \\ &+ \frac{1}{2} \left( \beta_1 x^2 + \beta_2 x^{\frac{2}{3}} \right) \sum_{a=4}^7 \Omega_a^2 + \frac{\sqrt{3}}{2} \Omega_8 \\ &- \left( s_1 x^2 + s_2 x^{\frac{2}{3}} + \frac{4}{9} s_3 \dot{x}^2 \right) (1 - D_{88}) . \end{aligned} \quad (9)$$

Here we have introduced the angular velocities  $A^\dagger \dot{A} = (i/2) \sum_{a=1}^8 \lambda_a \Omega_a$  as well as the adjoint representation  $D_{ab} = (1/2) \text{tr}(\lambda_a A \lambda_b A^\dagger)$ . A term linear in  $\dot{x}$ , which would originate from flavor symmetry breaking terms, has been omitted because the matrix elements of the associated  $SU(3)$  operators vanish when properly accounting for Hermiticity in the process of quantization [32]. The expressions for the constants  $a_1, \dots, s_3$  are functionals of the profile functions  $F(r)$  and  $\sigma_0(r)$ . In refs. [18, 19] their analytic expressions and numerical values are given. We also refer to these papers for a discussion on the approximate treatment of contributions from  $\mathcal{L}_{\text{SB}}$  that are quadratic in the angular velocities and an estimate of the minor influence of the  $s_3$ -term that will henceforth be omitted for

simplification. As far as the rotational collective coordinates are concerned, the Lagrangian (9) contains only a single flavor symmetry breaking operator,  $1 - D_{88}$ . Many other soliton model analyses consider up to three such operators [4, 17, 21–23]. Rather than determining the coefficients of these operators in an actual soliton model calculation they are frequently adjusted to baryon properties. The additional operators are subleading in  $1/N_C$  and do not emerge in models that have time derivatives appearing only with even powers in the flavor symmetry breaking terms of the Lagrangian, such as in eq. (3). On the other hand vector meson ( $V_\mu \partial^\mu U U^\dagger$ ) or chiral quark ( $\bar{\Psi} \gamma_\mu \partial^\mu \Psi$ ) model extensions may lead to such subleading operators. Explicit calculations for the coefficients of these operators in appropriate models indicate [12], however, that the inclusion of such subleading operators affects the resulting mass spectrum only slightly.

We are now in the position to canonically quantize the collective coordinates by constructing the Hamiltonian from the Lagrangian (9). For convenience we first define

$$\begin{aligned} m &= m(x) = \frac{8}{9} (a_1 + a_2 x^{-\frac{4}{3}}) , \\ b &= b(x) = b_1 x^{\frac{2}{3}} + b_2 x^{-\frac{2}{3}} + b_3 x^2 , \\ \alpha &= \alpha(x) = \alpha_1 x^2 + \alpha_2 x^{\frac{2}{3}} , \\ \beta &= \beta(x) = \beta_1 x^2 + \beta_2 x^{\frac{2}{3}} , \\ s &= s(x) = s_1 x^2 + s_2 x^{\frac{2}{3}} . \end{aligned} \quad (10)$$

The resulting Hamiltonian for the collective coordinates consists of a flavor-symmetric piece and a flavor symmetry breaking one. The flavor-symmetric part of the collective Hamiltonian

$$\begin{aligned} H_{\text{sym}} &= -\frac{1}{2\sqrt{m\alpha^3\beta^4}} \frac{\partial}{\partial x} \sqrt{\frac{\alpha^3\beta^4}{m}} \frac{\partial}{\partial x} + b \\ &+ \left( \frac{1}{2\alpha} - \frac{1}{2\beta} \right) \mathbf{J}^2 + \frac{1}{2\beta} C_2 - \frac{3}{8\beta} + s \end{aligned} \quad (11)$$

contains the collective rotations  $A$  only through their canonical momenta,  $R_a = -\frac{\partial L}{\partial \Omega_a}$ . These are the spin operator,  $\mathbf{J}^2 = \sum_{i=1}^3 R_i^2$ , and the quadratic Casimir operator,  $C_2 = \sum_{a=1}^8 R_a^2$ , of  $SU_F(3)$ . These operators are diagonal for a definite  $SU(3)$  representation,  $\mu$ . Due to the hedgehog structure of the static soliton configuration that is parameterized by  $U_0$  and  $\sigma_0$ , the allowed representations must contain at least one state with identical spin and isospin. In addition, the constraint  $R_8 = -\sqrt{3}/2$ , that originates from the Wess-Zumino term (6), requires this state to possess unit hypercharge and only permits states with half-integer spin [1, 11, 12]. Then the allowed  $SU(3)$  representations are  $\mu = \mathbf{8}$ ,  $\mu = \overline{\mathbf{10}}$ ,  $\mu = \mathbf{27}$ ,  $\mu = \overline{\mathbf{35}}$ , etc. for spin  $J = \frac{1}{2}$  and  $\mu = \mathbf{10}$ ,  $\mu = \mathbf{27}$ ,  $\mu = \mathbf{35}$ ,  $\mu = \overline{\mathbf{35}}$ , etc. for spin  $J = \frac{3}{2}$ . When we substitute the quantum numbers  $J(J+1)$  and  $C_2(\mu)$  ( $C_2(\mathbf{8}) = 3$ ,  $C_2(\mathbf{10}) = C_2(\overline{\mathbf{10}}) = 6$ ,  $C_2(\mathbf{27}) = 8$ , etc.),  $H_{\text{sym}}$  is as simple as an ordinary second-order differential operator in the monopole mode  $x$ . This

Schrödinger equation can straightforwardly be solved numerically. For definiteness we denote the eigenvalues by  $\mathcal{E}_{\mu, n_\mu}$  and the corresponding eigenstates by  $|\mu, n_\mu\rangle$ , where  $n_\mu$  labels the monopole (or radial) excitations for a prescribed  $SU(3)$  representation,  $\mu$ . According to the above discussion the eigenstates factorize, *i.e.*  $|\mu, n_\mu\rangle = |\mu\rangle|n_\mu\rangle$ . In this language the nucleon corresponds to  $|\mathbf{8}, 0\rangle$  while the first radially excited state, which is commonly identified with the Roper (1440) resonance, would be  $|\mathbf{8}, 1\rangle$ . We are interested in the role of states like  $|\overline{\mathbf{10}}, n_{\overline{\mathbf{10}}}\rangle$  since in particular this tower contains the state with the quantum numbers of the exotic pentaquarks  $\Theta^+$  and  $\Xi_{3/2}$ . Their partners with spin  $J = 3/2$  are contained in  $|\mathbf{27}, n_{\mathbf{27}}\rangle$ .

In the second step we include the symmetry breaking part of the Hamiltonian obtained canonically from the Lagrangian (9). We utilize the above-constructed states  $|\mu, n_\mu\rangle$  as a basis to diagonalize the complete Hamiltonian matrix

$$H_{\mu, n_\mu; \mu', n'_\mu} = \mathcal{E}_{\mu, n_\mu} \delta_{\mu, \mu'} \delta_{n_\mu, n'_\mu} - \langle \mu | D_{88} | \mu' \rangle \langle n_\mu | s(x) | n'_\mu \rangle. \quad (12)$$

The flavor part of these matrix elements is computed using  $SU(3)$  Clebsch-Gordan coefficients while the radial part is calculated numerically using the appropriate eigenstates of (11). Of course, this can be done for each isospin multiplet separately, *i.e.* flavor quantum numbers are not mixed. We stress that we *exactly* diagonalize the complete Hamiltonian,  $H_{\text{sym}} - s(x)D_{88}$  rather than approximating the eigenvalues and eigenstates in form of a perturbation in flavor symmetry breaking. The physical baryon states  $|B, m\rangle$  are finally expressed as linear combinations of the eigenstates of the symmetric part

$$|B, m\rangle = \sum_{\mu, n_\mu} C_{\mu, n_\mu}^{(B, m)} |\mu, n_\mu\rangle. \quad (13)$$

The corresponding eigenenergies are denoted by  $E_{B, m}$ . The nucleon  $|N, 0\rangle$  is then identified as the lowest-energy solution with the associated quantum numbers, while the Roper is obtained as the next state ( $|N, 1\rangle$ ) in the same spin-isospin channel. Turning to the quantum numbers of the  $\Lambda$  provides not only the energy  $E_{\Lambda, 0}$  and wave function  $\langle A, x | \Lambda, 0 \rangle$  of this hyperon but also the analogous quantities for the radially excited  $\Lambda$ 's:  $E_{\Lambda, m}$  and  $\langle A, x | \Lambda, m \rangle$  with  $m \geq 1$ . These calculations are repeated for the other spin-isospin channels yielding the spectrum not only of the ground-state  $\frac{1}{2}^+$  and  $\frac{3}{2}^+$  baryons but also their monopole excitations. Of course, flavor symmetry breaking couples all possible  $SU(3)$  representations as  $\langle \mu | D_{88} | \mu' \rangle$  is not diagonal in  $\mu$  and  $\mu'$ . It also mixes various monopole excitations of the basis states  $|\mu, n_\mu\rangle$ . When diagonalizing (12) we consider the basis built by the representations  $\mathbf{8}, \overline{\mathbf{10}}, \mathbf{27}, \overline{\mathbf{35}}, \mathbf{64}, \overline{\mathbf{81}}, \mathbf{125}, \overline{\mathbf{154}}$  for the  $\frac{1}{2}^+$  baryons and  $\mathbf{10}, \mathbf{27}, \mathbf{35}, \overline{\mathbf{35}}, \mathbf{64}, \mathbf{28}, \mathbf{81}, \overline{\mathbf{81}}, \mathbf{125}, \overline{\mathbf{80}}, \mathbf{154}, \overline{\mathbf{254}}$  for the  $\frac{3}{2}^+$  baryons. For the breathing degree of freedom we include basis states which are up to 4 GeV above the ground states of the flavor-symmetric piece (11), *i.e.*  $|\mathbf{8}, 0\rangle$  and  $|\mathbf{10}, 0\rangle$  for the  $\frac{1}{2}^+$  and  $\frac{3}{2}^+$  baryons,

respectively. This seems to be sufficient to get acceptable convergence when diagonalizing (12). It should be noted that not all of the above  $SU(3)$  representations appear in each isospin channel. For example, there are no  $\Lambda$ - and  $\Xi$ -type states in the antidecuplet,  $\overline{\mathbf{10}}$ .

## 4 Results

We expect three categories of model results. In the first place we have the ordinary low-lying  $J = \frac{1}{2}$  and  $J = \frac{3}{2}$  baryons together with their monopole excitations. Without flavor symmetry breaking these would be pure-octet and -decuplet states. In the second place we have  $J = \frac{1}{2}$  states that are dominantly members of the antidecuplet. Those that are non-exotic mix with octet baryons and their monopole excitations. We are particularly interested in dominantly antidecuplet baryons ( $\Theta^+$  and  $\Xi_{3/2}$ ) that do not have partners with identical quantum numbers in the octet. These antidecuplet baryons cannot be constructed as three-quark composites and are purely exotic. Third, we have  $J = \frac{3}{2}$  baryons that originate from the  $\mathbf{27}$ -plet, cf. lower panel of fig. 1. Not all of them are exotic, as the  $\Delta_{27}, \Sigma_{27}^*$  and  $\Xi_{27}^*$  have partners in the decuplet. Finally there are also  $J = \frac{1}{2}$  baryons in the  $\mathbf{27}$ -plet. They are heavier than the  $J = \frac{3}{2}$  members of the  $\mathbf{27}$ -plet and will therefore not be studied here. Note that we will consider only mass differences (with respect to the nucleon) because they are expected to be more reliably predicted in chiral soliton models than absolute masses [33].

### 4.1 Ordinary baryons and their monopole excitations

In table 1 the predictions for the mass differences with respect to the nucleon of the eigenstates of the full Hamiltonian (12) are shown for two values of the Skyrme parameter  $e$ . These results were already reported earlier [6, 18, 19]. For completeness and later comparison with the interesting  $\mathbf{8} \oplus \overline{\mathbf{10}}$  picture of refs. [16, 17], we also discuss those results here.

The agreement with the experimental data is quite astonishing, not only for the ground state but also for the radial excitations. Only the prediction for the Roper resonance ( $|N, 1\rangle$ ) is a bit on the low side when compared to the empirical Breit-Wigner mass but agrees reasonably well with the estimated pole position of that resonance. This is common for the breathing mode approach in soliton models [30, 31]. For other nucleon (and  $\Delta$ ) resonances the discrepancy between the Breit-Wigner mass and the pole position is much smaller, of the order of 20 MeV or less [34]. Thus, there is no need to distinguish between them. As far as data are available and the quantum numbers are definite, the other first-excited states are quite well reproduced. For the  $\frac{1}{2}^+$  baryons the energy eigenvalues for the second excitations overestimate the corresponding empirical data somewhat. However, the pattern  $M(|N, 2\rangle) < M(|\Sigma, 2\rangle) < M(|\Lambda, 2\rangle)$  is reproduced if  $|\Sigma, 2\rangle$  is identified with the single-star resonance  $\Sigma(1770)$  that is

**Table 1.** The mass differences with respect to the nucleon of the eigenstates of the Hamiltonian (12). The notation for the states appearing in this table is defined in eq. (13). All numbers are in MeV. Experimental data are taken from [34], if available. Unless otherwise noted, the data from [34] refer to four- and three-star resonances. For the Roper resonance ( $N(1440)$ ) we list both the Breit-Wigner (BW) mass and the pole position (PP) estimate of ref. [34]. The experimental states furnished with “?” in the  $|\Xi, 1\rangle$  column are potential isospin- $\frac{1}{2}$  candidates whose spin-parity quantum numbers are not yet determined [34].

$B$	$m = 0$			$m = 1$			$m = 2$		
	$e = 5.0$	$e = 5.5$	expt.	$e = 5.0$	$e = 5.5$	expt.	$e = 5.0$	$e = 5.5$	expt.
N	Input			413	445	501 BW 426 PP	836	869	771
$\Lambda$	175	173	177	657	688	661	1081	1129	871
$\Sigma$	284	284	254	694	722	721	1068	1096	831 (*) 941 (**)
$\Xi$	382	380	379	941	971	751 1011 (?)	1515	1324	–
$\Delta$	258	276	293	640	680	661	974	1010	981
$\Sigma^*$	445	460	446	841	878	901	1112	1148	1141
$\Xi^*$	604	617	591	1036	1068	–	1232	1269	–
$\Omega$	730	745	733	1343	1386	–	1663	1719	–

**Table 2.** Masses of the eigenstates of the Hamiltonian (12) for the exotic baryons  $\Theta^+$  and  $\Xi_{3/2}$ . Experimental data are the average of refs. [2] for  $\Theta^+$  and the NA49 result for  $\Xi_{3/2}$  [3]. We also compare the predictions for the ground state ( $m = 0$ ) to the treatment of ref. [21] that does not quantize the monopole mode. All energies are in GeV.

$B$	$m = 0$				$m = 1$		
	$e = 5.0$	$e = 5.5$	expt.	WK [21]	$e = 5.0$	$e = 5.5$	expt.
$\Theta^+$	1.57	1.59	$1.537 \pm 0.010$	1.54	2.02	2.07	–
$\Xi_{3/2}$	1.89	1.91	$1.862 \pm 0.002$	1.78	2.29	2.33	–

about 830 MeV heavier than the nucleon. The predicted  $\Sigma$ - and  $\Lambda$ -type states with  $m = 2$  are about 100–200 MeV too high. It is worth noting that in the nucleon channel the  $m = 3$  state is predicted to be only about 40 MeV higher than the  $m = 2$  state, *i.e.* still within the regime where the model is assumed to be applicable. This is interesting because empirically it is suggestive that there might exist more than only one resonance in the concerned energy region [35]. For the  $\frac{3}{2}^+$  baryons with  $m = 2$  the agreement with data is even better, on the 3% level. The particle data group [34] lists two “three-star” isospin- $\frac{1}{2}$   $\Xi$ -resonances whose spin-parity quantum numbers are not yet determined at 751 and 1011 MeV above the nucleon. Turning to absolute masses these are  $\Xi(1690)$  and  $\Xi(1950)$ , respectively. The present model suggests that the latter is indeed  $J^\pi = \frac{1}{2}^+$ , while the former seems to belong to a different channel. For example, the sizable mass difference between the lowest-lying ( $m = 0$ )  $\Sigma$  and  $\Xi$  motivates the speculation that the  $\Xi(1690)$  should be considered as a vibrational ( $\hbar\omega$ ) excitation of the octet  $\Xi$  and that it originates from a multiplet with  $N(1520)$ ,  $\Lambda(1520)$  and  $\Sigma(1580)$ . These are  $D$ -wave resonances with  $J^\pi = \frac{3}{2}^-$  that are prominently observed in scattering calculations off the soliton [36]. On the other hand, the established  $D$ -wave  $J^\pi = \frac{3}{2}^-$  state  $\Xi(1820)$  may be associated with a multiplet formed with  $N(1700)$ ,  $\Lambda(1690)$ ,  $\Sigma(1670)$ . Such a picture is somewhat appealing as both  $J^\pi = \frac{3}{2}^-$  octets would have (almost) degenerate nucleon and  $\Lambda$  states. Here we finally note that the model state  $|\Xi, 1\rangle$  is dominated by the first radial excitation in the octet and the ground state in the **27**-plet, as  $C_{8,1}^{(\Xi,1)2} \approx C_{27,0}^{(\Xi,1)2} \approx 0.4$ .

On the whole, the present model gives fair agreement with the available data. This certainly supports the picture of coupled monopole and rotational modes. The important message is that three flavor chiral models do *not* predict any novel states in the energy regime between 1 and 2 GeV in the non-exotic channels as a consequence of including higher-dimensional  $SU_F(3)$  flavor representations.

## 4.2 Exotic baryons from the antidecuplet

The  $\overline{\mathbf{10}}$  representation contains two states that possess quantum numbers that cannot be constructed as three-quark composites, the  $\Theta^+$  and the  $\Xi_{3/2}$ . The model prediction for these states are listed in table 2 and compared to available data [2, 3]. As for the non-exotic baryons, we have computed the respective mass differences to the nucleon and added the experimental nucleon mass to set the overall mass scale. We also compare to a chiral soliton model calculation [21] that does not include a dynamical treatment of the monopole excitation. In that calculation parameters have been tuned to reproduce the mass of the lightest exotic pentaquark,  $\Theta^+$ . The inclusion of the monopole excitation increases the mass of the  $\Xi_{3/2}$  slightly and brings it closer to the empirical value. We furthermore note that the first prediction [4] for the mass of the  $\Xi_{3/2}$  that was based on identifying  $N(1710)$  with the nucleon-like state in the antidecuplet resulted in a far too large mass of 2070 MeV. There are other chiral soliton model studies of the pentaquarks of the antidecuplet. However, those either take  $M_{\Xi_{3/2}}$  as input [17, 22], adopt

**Table 3.** Predicted masses of the eigenstates of the Hamiltonian (12) for the exotic  $J = \frac{3}{2}$  baryons with  $m = 0$  and  $m = 1$  that originate from the **27**-plet. The corresponding hypercharge ( $Y$ ) and isospin ( $I$ ) quantum numbers are listed. We also compare the  $m = 0$  case to treatments without breathing mode quantization of refs. [21–23]. The results of ref. [22] have been confirmed in ref. [10]. All numbers are in GeV.

$B$	$Y$ $I$		$m = 0$					$m = 1$	
			$e = 5.0$	$e = 5.5$	WK [21]	BFK [22]	WM [23]	$e = 5.0$	$e = 5.5$
$\Theta_{27}$	2	1	1.66	1.69	1.67	1.60	1.60	2.10	2.14
$N_{27}$	1	1/2	1.82	1.84	1.76	–	1.73	2.28	2.33
$\Lambda_{27}$	0	0	1.95	1.98	1.86	–	1.86	2.50	2.56
$\Gamma_{27}$	0	2	1.70	1.73	1.70	1.70	1.68	2.12	2.17
$\Pi_{27}$	–1	3/2	1.90	1.92	1.84	1.88	1.87	2.35	2.40
$\Omega_{27}$	–2	1	2.08	2.10	1.99	2.06	2.07	2.54	2.59

the assumptions of ref. [4] or are less predictive because the model parameters are allowed to vary considerably [10]. In any event, it is desirable to independently confirm the NA49 analysis that led to  $M_{\Xi_{3/2}} = 1.862 \pm 0.002$ .

In all cases where comparison with data is possible, we observe that without any fine-tuning the model prediction is only about 30–50 MeV higher than the data. In view of the approximative nature of the model this level of agreement should be viewed as good. In particular the mass difference between the two potentially observed exotics is reproduced within 10 MeV, cf. table 2. Notably, both the empirical data as well as the present-model calculation for this mass difference are slightly above the upper limit  $M_{\Xi_{3/2}} - M_{\Theta^+} < 299$  MeV obtained in a quark model with a color magnetic interaction [37].

### 4.3 Baryons from the 27-plet

The **27** dimensional representation allows for both baryons with spin  $J = \frac{1}{2}$  and  $J = \frac{3}{2}$ . Since  $\alpha(x) > \beta(x)$  [18,19] the  $J = \frac{3}{2}$  baryons will be lighter according to the Hamiltonian (11,12). We will therefore only consider the  $J = \frac{3}{2}$  baryons. As is shown in fig. 1, the **27**-plet contains states with the quantum numbers of the baryons that are also contained in the decuplet of the low-lying  $J = \frac{3}{2}$  baryons:  $\Delta$ ,  $\Sigma^*$  and  $\Xi^*$ . Under flavor symmetry breaking these states of the decuplet and **27**-plet mix with each other as well as with the respective monopole excitations. Stated otherwise, the  $\Delta$ ,  $\Sigma^*$  and  $\Xi^*$  states that emerge from the **27**-plet have been already considered when diagonalizing the full Hamiltonian (12) for the states with quantum numbers of the low-lying  $J = \frac{3}{2}$  baryons. Eventually these members of the **27**-plet represent the dominant amplitude in the exact eigenstates  $|\Delta, 1\rangle$  and/or  $|\Delta, 2\rangle$  and similarly for  $\Sigma^*$  and  $\Xi^*$ . The corresponding eigenvalues are displayed and compared to available data in table 1. In table 3 we present the model predictions for the  $J = \frac{3}{2}$  baryons that emerge from the **27**-plet but do not have partners in the lower-dimensional representations. Again, we have used the experimental nucleon mass to set the overall mass scale. Essentially the masses displayed in table 3 are model predictions for resonances that are yet to be observed. Nevertheless we note that the particle data group [34] lists two states with the quantum numbers of  $N_{27}$  and  $\Lambda_{27}$  at 1.72 and 1.89 GeV, respectively, that fit

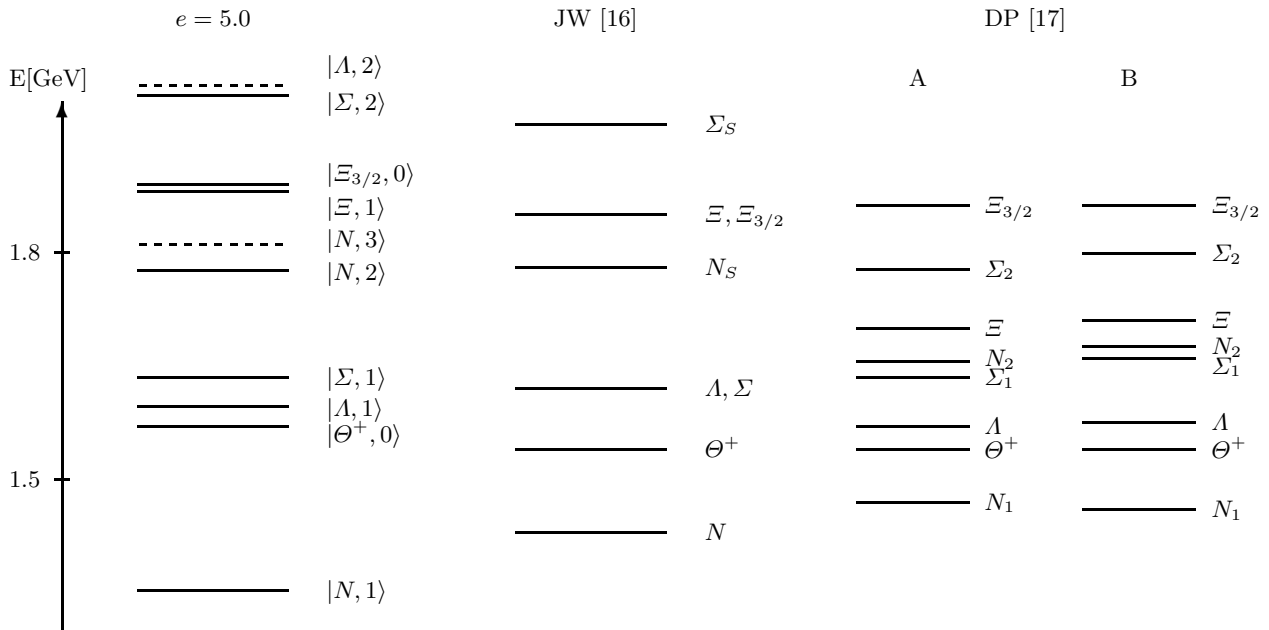
reasonably well into the model calculation. In all channels the  $m = 1$  states turn out to be about 500 MeV heavier than the exotic ground states. When combined with the first-excited states of  $\Delta$ ,  $\Sigma^*$ , and  $\Xi^*$  that are listed under  $m = 1$  in table 1 it is interesting to observe that the masses of states that are degenerate in hypercharge decrease with isospin, that is  $M_{|\Delta,1\rangle} < M_{|N_{27,0}\rangle}$ ,  $M_{|\Gamma_{27,0}\rangle} < M_{|\Sigma^*,1\rangle} < M_{|\Lambda_{27,0}\rangle}$ ,  $M_{|\Pi_{27,0}\rangle} < M_{|\Xi^*,1\rangle}$ , and  $M_{|\Omega_{27,0}\rangle} < M_{|\Omega,1\rangle}$ .

The comparison with treatments [10,21–23] that do not quantize the monopole degree of freedom indicates that for the exotic pentaquark states that do not have partners in lower-dimensional  $SU_F(3)$  representations, the mixing of rotational and monopole modes is a minor effect and can eventually be discarded. The treatment of ref. [21] is similar to the present one in the sense that flavor symmetry breaking is treated to all orders when diagonalizing the Hamiltonian for the collective degrees of freedom. On the other hand, in refs. [10,22,23] a first-order approximation has been adopted. Not surprisingly, the model calculations that do not quantize the monopole excitations find masses for the non-exotic states  $\Delta_{27}$ ,  $\Sigma_{27}^*$  and  $\Xi_{27}^*$ , that lie in between the  $m = 1$  and  $m = 2$  eigenvalues of  $\Delta$ ,  $\Sigma^*$  and  $\Xi^*$ , cf. table 1. For example, the authors of ref. [21] predict  $M_{\Delta_{27}} \approx 1.78$ ,  $M_{\Sigma_{27}^*} \approx 1.86$  and  $M_{\Xi_{27}^*} \approx 1.95$ . Only for  $e = 5.5$  both the  $m = 1$  ( $1.07 + 0.94 = 2.01$  GeV) and the  $m = 2$  ( $1.27 + 0.94 = 2.21$  GeV) for  $\Xi^*$  are slightly above that prediction for  $\Xi_{27}^*$ . This may be connected to the observation that we presently also find a somewhat heavier  $\Xi_{3/2}$  than in ref. [21], cf. table 2.

The statement that the dynamical treatment of the monopole degree of freedom does not significantly alter the mass eigenvalues of the exotic baryons is also supported by the observation that those states acquire their dominant contributions to the wave functions from the **27**-plet. For a convenient discussion thereof we present in table 4 the sums

$$P_\mu = \sum_{n_\mu} \left[ C_{\mu, n_\mu}^{(B, m)} \right]^2 \quad (14)$$

of the squared amplitudes that are defined in eq. (13). The  $P_\mu$  are to be interpreted as the probability to find a state in a given irreducible representation,  $\mu$ , of  $SU_F(3)$ . As is already known for the  $J^\pi = \frac{1}{2}^+$  states [6, 18, 19, 24], the lowest-lying state in a channel with given hypercharge and



**Fig. 2.** Comparison of the predicted spectra in various models for pentaquarks with  $J = \frac{1}{2}$ . The present model is labeled by  $e = 5.0$ , the parameter chosen for presentation. Two additional states that fall into the depicted energy range are indicated by dashed lines. For the non-exotic states the model results are taken from table 1 with the physical nucleon mass added. JW denotes a calculation within the Jaffe-Wilczek diquark model [16] with the parameters  $M_0 = 1.44 \text{ GeV}$ ,  $m_s = 0.11 \text{ GeV}$  and  $\alpha = 0.06 \text{ GeV}$  substituted into their mass formula. In columns A and B the results of the two scenarios of the Diakonov-Petrov approach [17] are shown. We have adopted the notation used in the respective papers.

isospin is strongly dominated by the state from the lowest allowed  $SU_F(3)$  representation, *e.g.* the nucleon state is dominantly  $|\mathbf{8}, 0\rangle$ . On the other hand, the first-excited state turns out to be a complicated linear combination of all possible basis states, again a pattern also observed for the  $J = \frac{1}{2}$  eigenstates of the Hamiltonian (12). In the following section we will discuss the mixing pattern for the  $J = \frac{1}{2}$  states in more detail.

## 5 Comparison with the $\mathbf{8} \oplus \overline{\mathbf{10}}$ picture

In the present approach to compute the masses of pentaquarks there exists an important and obvious interplay between the radially excited octet baryons and the ground-state baryons in the antidecuplet. We expect significant mixing for states in the 1.3 to 1.8 GeV energy regime because the eigenenergies  $\mathcal{E}_{\mathbf{8},1}$  and  $\mathcal{E}_{\overline{\mathbf{10}},0}$  of the basis states are not only about 0.5 GeV above  $\mathcal{E}_{\mathbf{8},0}$  but also quite close together. For example  $\mathcal{E}_{\mathbf{8},1} - \mathcal{E}_{\overline{\mathbf{10}},0} \approx 18 \text{ MeV}$  for  $e = 5.0$ . Hence, even if the associated symmetry breaking matrix elements  $\langle \mu, n_\mu | s D_{88} | \mu', n_{\mu'} \rangle$  in eq. (12) were small, we expect large mixing effects. The eigenenergies of other basis states are several hundred MeV away and thus it is plausible that the  $J^\pi = \frac{1}{2}^+$  baryons in the energy regime between 1.3 and 1.8 GeV may be viewed as members of the direct sum  $\mathbf{8} \oplus \overline{\mathbf{10}}$  of the  $SU_F(3)$  representations, at least approximately. In such a  $\mathbf{8} \oplus \overline{\mathbf{10}}$  scenario the  $\Lambda$  and  $\Xi$  resonances are pure octet while  $\Theta^+$

and  $\Xi_{3/2}$  are pure antidecuplet. Mixing can only occur for nucleon and  $\Sigma$ -type states. There are two approaches to arrange the excited baryons within such a direct sum. In ref. [16] a diquark picture has been adopted that leads to an ideal mixing between baryons of identical quantum numbers within the  $\mathbf{8}$  and  $\overline{\mathbf{10}}$  representations such that the eigenstates have minimal or maximal strangeness content. For example, the octet nucleon and the antidecuplet nucleon ( $N'$  in fig. 1) mix to build eigenstates with the quark structure  $uud(\bar{u}u)$  and  $uud(\bar{s}s)$ , modulo the isospin partners. Then the baryon mass formula essentially counts the number of strange quarks and antiquarks that are contained in the resonance. In ref. [17] the  $\mathbf{8} \oplus \overline{\mathbf{10}}$  decomposition was taken as starting point. For these octet and antidecuplet states the pattern of flavor symmetry breaking was adopted which results from a first-order calculation in soliton models. Those authors additionally assumed that  $\Xi(1690)$  —whose spin-parity quantum numbers are yet to be determined [34]— was the octet partner of  $\Lambda(1600)$  to estimate the octet mass parameters and fixed the antidecuplet mass parameters from  $\Theta^+(1535)$  and  $\Xi_{3/2}(1862)$ . With that input they computed the mixing angle between the  $\mathbf{8}$  and  $\overline{\mathbf{10}}$  representations and predicted so far unobserved nucleon and  $\Sigma$ -resonances in the 1650–1810 MeV region. In fig. 2 we compare the mass spectra of these two  $\mathbf{8} \oplus \overline{\mathbf{10}}$  scenarios with the present-model calculation. Surprisingly the spectra obtained in the current-model calculation and that of the Jaffe-Wilczek ideal mixing scenario are very similar, at least qualitatively. The most apparent



**Table 4.** Mixing pattern for the  $J^\pi = \frac{3}{2}^+$  states. In case of  $\Delta$ ,  $\Sigma^*$ ,  $I_{27}$ , and  $\Pi_{27}$  the notations  $\mathbf{35}_1$  and  $\mathbf{35}_2$  denote unspecified orthonormal linear combinations of the corresponding states from  $\mathbf{35}$  and  $\overline{\mathbf{35}}$ . In all other cases  $\mathbf{35}_1$  and  $\mathbf{35}_2$  equal  $\mathbf{35}$  and  $\overline{\mathbf{35}}$ , respectively. Listed are the probabilities  $P_\mu$  defined in eq. (14) for  $e = 5.0$ .

$B, m$	$\mu = \mathbf{10}$	$\mu = \mathbf{27}$	$\mu = \mathbf{35}_1$	$\mu = \mathbf{35}_2$	$\mu = \mathbf{64}$
$ \Delta, 0\rangle$	0.60	0.30	0.04	0.03	0.02
$ \Delta, 1\rangle$	0.54	0.22	0.06	0.09	0.05
$ \Delta, 2\rangle$	0.19	0.61	0.06	0.09	0.03
$ \Delta, 3\rangle$	0.57	0.10	0.04	0.16	0.04
$ \Sigma^*, 0\rangle$	0.73	0.18	0.06	0.01	0.01
$ \Sigma^*, 1\rangle$	0.47	0.27	0.11	0.05	0.07
$ \Sigma^*, 2\rangle$	0.14	0.67	0.11	0.03	0.02
$ \Xi^*, 0\rangle$	0.87	0.07	0.05	–	0.01
$ \Xi^*, 1\rangle$	0.31	0.47	0.13	–	0.05
$ \Xi^*, 2\rangle$	0.23	0.52	0.19	–	0.01
$ \Omega, 0\rangle$	0.97	–	0.03	–	0.00
$ \Omega, 1\rangle$	0.55	–	0.39	–	0.06
$ \Omega, 2\rangle$	0.50	–	0.36	–	0.12
$ \Theta_{27}, 0\rangle$	–	0.76	–	0.16	0.05
$ \Theta_{27}, 1\rangle$	–	0.54	–	0.25	0.08
$ N_{27}, 0\rangle$	–	0.87	–	0.07	0.06
$ N_{27}, 1\rangle$	–	0.49	–	0.20	0.19
$ A_{27}, 0\rangle$	–	0.93	–	–	0.06
$ A_{27}, 1\rangle$	–	0.56	–	–	0.37
$ I_{27}, 0\rangle$	–	0.73	0.09	0.10	0.06
$ I_{27}, 1\rangle$	–	0.52	0.14	0.11	0.37
$ \Pi_{27}, 0\rangle$	–	0.82	0.11	0.00	0.05
$ \Pi_{27}, 1\rangle$	–	0.46	0.25	0.00	0.19
$ \Omega_{27}, 0\rangle$	–	0.90	–	0.06	0.04
$ \Omega_{27}, 1\rangle$	–	0.29	–	0.41	0.16

similarity is the (almost) degeneracy of  $\Xi_{3/2}$  and the first-excited  $\Xi$ . The model of ref. [16] also has degenerate  $\Lambda$  and  $\Sigma$  states. In the present approach such a degeneracy is also indicated but not very pronounced. Furthermore, both treatments yield the second-excited  $\Sigma$  way above the  $\Xi_{3/2}$  as well as a large gap between the first-excited  $\Sigma$  and the second-excited nucleon. This is somewhat different in the analysis of ref. [17]: Most obvious is the large gap between  $\Xi$  and  $\Xi_{3/2}$  with the second-excited  $\Sigma$  sitting in between. For that result it was crucial to assume that  $\Xi(1690)$  is in the same  $SU_F(3)$  multiplet as the  $\Lambda(1600)$ . While  $\Lambda(1600)$  has  $J^\pi = \frac{1}{2}^+$ , the spin-parity quantum numbers of  $\Xi(1690)$  are not yet determined experimentally. If indeed  $\Xi(1690)$  were the partner of the  $\Lambda(1600)$ , one would expect to also obtain such a state in the  $J^\pi = \frac{1}{2}^+$  channel in the quark model calculation of ref. [38]. However, in that calculation the first-excited  $\Xi$  with  $J^\pi = \frac{1}{2}^+$  shows up significantly higher, at about 1850 MeV, *i.e.* similar to the present prediction for  $|\Xi, 1\rangle$ , cf. table 1. As already discussed in sect. 3.3, it seems more plausible to assign  $J^\pi = \frac{3}{2}^-$  to  $\Xi(1690)$  and  $J^\pi = \frac{1}{2}^+$  to  $\Xi(1950)$ . From tables 4 and 5 we recognize a similarity between the structure of the nucleon and  $\Delta$  states. The first-excited state ( $m = 1$ ) resides mostly in the lowest possible dimensional  $SU_F(3)$  representation, *i.e.* a monopole exci-

**Table 5.** Mixing pattern for the low-lying  $J^\pi = \frac{1}{2}^+$  baryons and the states shown in fig. 2. Listed are the probabilities  $P_\mu$  defined in eq. (14) for  $e = 5.0$ .

$B, m$	$\mu = \mathbf{8}$	$\mu = \overline{\mathbf{10}}$	$\mu = \mathbf{27}$	$\mu = \overline{\mathbf{35}}$	$\mu = \mathbf{64}$
$ N, 0\rangle$	0.87	0.06	0.05	0.01	0.00
$ N, 1\rangle$	0.59	0.15	0.16	0.05	0.03
$ N, 2\rangle$	0.12	0.68	0.10	0.05	0.04
$ N, 3\rangle$	0.57	0.17	0.08	0.12	0.03
$ \Lambda, 0\rangle$	0.93	–	0.06	–	0.00
$ \Lambda, 1\rangle$	0.58	–	0.34	–	0.07
$ \Lambda, 2\rangle$	0.58	–	0.23	–	0.17
$ \Sigma, 0\rangle$	0.88	0.08	0.04	0.01	0.00
$ \Sigma, 1\rangle$	0.37	0.33	0.15	0.10	0.03
$ \Sigma, 2\rangle$	0.11	0.69	0.13	0.04	0.04
$ \Xi, 0\rangle$	0.96	–	0.04	–	0.00
$ \Xi, 1\rangle$	0.49	–	0.42	–	0.07
$ \Theta^+, 0\rangle$	–	0.85	–	0.14	–
$ \Xi_{3/2}, 0\rangle$	–	0.76	0.12	0.10	0.01

tation of the ground state. The  $m = 2$  state is dominated by the member of the next-to-lowest possible dimensional  $SU_F(3)$  representation while  $m = 3$  again is in the same multiplet as the ground and originates from the second monopole excitation.

Although the spectrum in the present model turns out to be similar to the Jaffe-Wilczek scenario, the mixing pattern is considerably more complicated than an ideal direct sum  $\mathbf{8} \oplus \overline{\mathbf{10}}$ . In table 5 we give the mixing for the states depicted in fig. 2 in form of the probabilities  $P_\mu$  that have been defined in eq. (14). In the  $\mathbf{8} \oplus \overline{\mathbf{10}}$  scenarios of refs. [16, 17] both the  $|\Lambda, 1\rangle$  and the  $|\Xi, 1\rangle$  would be pure octet states. In the present-model calculation we find, however, that there is significant admixture of the partners from the  $\mathbf{27}$ -plet, at the order of 40% in the squared amplitude,  $P_\mu$ . Furthermore the states  $|N, 1\rangle$  and  $|N, 2\rangle$  as well as  $|\Sigma, 1\rangle$  and  $|\Sigma, 2\rangle$  are not simple linear combinations of the corresponding octet and antidecuplet states but also contain sizable contributions from their partners in the  $\mathbf{27}$ -plet. In ref. [16] the mixing pattern for the  $\mathbf{8}$  and  $\overline{\mathbf{10}}$  states is ideal for the strangeness content, *i.e.*  $N$  and  $\Sigma$  have minimal strangeness content while it is maximal for  $N_S$  and  $\Sigma_S$ . In order to further compare the present scenario we therefore estimate the strangeness content as the matrix element [18],

$$S(B, m) = \frac{1}{3} \langle B, m | [1 - D_{88}] | B, m \rangle \quad (15)$$

for the nucleon and  $\Sigma$  eigenstates of the Hamiltonian (12). In the soliton approach baryons naturally possess mesonic (quark-antiquark) clouds. It is therefore quite instructive to first compare the model results to the flavor-symmetric formulation. In that case the nucleon and  $\Sigma$  ground states are pure octet and the above matrix elements are simple Clebsch-Gordan coefficients:  $S(N, \mathbf{8}) = 23\%$  and  $S(\Sigma, \mathbf{8}) = 37\%$ . Similarly the antidecuplet states have  $S(N, \overline{\mathbf{10}}) = 29\%$  and  $S(\Sigma, \overline{\mathbf{10}}) = 33\%$ . This large value for the nucleon reflects a large  $\bar{s}s$  cloud that is easily excited when the strange quark is assumed to be (almost)

**Table 6.** Strangeness content in per cent as computed from eq. (15) for  $e = 5.0$ .

	$ N, 0\rangle$	$ N, 1\rangle$	$ N, 2\rangle$	$ \Sigma, 0\rangle$	$ \Sigma, 1\rangle$	$ \Sigma, 2\rangle$
$S$	16	14	19	30	23	29

massless, as is the case in the flavor-symmetric treatment. In table 6 we list the strangeness content for the three lowest nucleon and  $\Sigma$  states. As expected, the inclusion of flavor symmetry breaking, *i.e.* the increase of the strange-quark mass, leads to an overall reduction. Furthermore the lower-lying excited states in the nucleon and  $\Sigma$  channels possess smaller strangeness content than the second-excited states. This is in the logic of the ideal mixing pattern [16] but not quite as pronounced as to maximize or minimize the strangeness content. Remarkably we find that the strangeness content of the first-excited nucleon and  $\Sigma$ -type states is even less than that of the respective ground states.

## 6 Conclusion

We have analyzed the interplay between rotational and monopole excitations for the spectrum of pentaquarks in a chiral soliton model. The main issue in this approach has been the elevation of the scaling degree of freedom to a dynamical quantity which subsequently has been quantized canonically at the same footing as the (flavor) rotational modes. In this manner not only the ground states in individual irreducible  $SU_F(3)$  representations are eigenstates of the (flavor-symmetric part of the) Hamiltonian but also all their radial excitations. It turns out that the various rotational and monopole excitations mix via the flavor symmetry breaking term in this Hamiltonian. In determining the baryon spectrum we have treated flavor symmetry breaking exactly rather than only at first order; an approximation often performed [4, 10, 17, 22, 23]. Thus, even though the chiral soliton approach initiates from a flavor-symmetric formulation, it is quite well capable of accounting for large deviations thereof.

The spectrum of the low-lying  $\frac{1}{2}^+$  and  $\frac{3}{2}^+$  baryons is reasonably well reproduced. It is also important to note that the results for static properties such as magnetic moments and axial transition matrix elements that parameterize the amplitudes of semileptonic hyperon decays are in acceptable agreement with the empirical data [18]. This makes the model reliable to also study the spectrum of the excited states. Indeed there is a clear way to associate the model states with the observed baryon excitations; except maybe an additional P11 nucleon state although there exist analyses with such a resonance. Otherwise, this model calculation did not indicate the existence of any so far unobserved baryon state with quantum numbers of three-quark composites. The comparison with the empirical spectrum furthermore has led to the speculation that the so far undetermined spin-parity quantum numbers of the hyperon resonances  $\Xi(1690)$  and  $\Xi(1950)$  should be

$J^\pi = \frac{3}{2}^-$  and  $J^\pi = \frac{1}{2}^+$ , respectively. Lastly we have seen that the computed masses for the exotic  $\Theta^+$  and  $\Xi_{3/2}$  baryons nicely agree with the recent observation for these pentaquarks. The quality of these results is remarkable, after all at this stage the model contains no more adjustable parameter. The mass difference between mainly octet and mainly antidecuplet baryons thus is a prediction while it is an input quantity in most other approaches [4, 10, 21–23]. We are thus confident that the present predictions for the masses of the spin- $\frac{3}{2}$  pentaquarks are sensible as well and we roughly expect them between 1.6 and 2.1 GeV. It is known and obvious from the Hamiltonian that in chiral soliton model estimates the masses of states with otherwise identical quantum numbers decrease with spin. In the present calculation we have observed this pattern also for isospin.

Moreover, this approach almost naturally yielded a classification for the next-to-lowest-lying  $J^\pi = \frac{1}{2}^+$  baryon states that is similar to the  $\mathbf{8} \oplus \overline{\mathbf{10}}$  scenario found in a diquark model for the pentaquark baryons. Even though our soliton model spectrum for these next-to-lowest-lying  $J^\pi = \frac{1}{2}^+$  states qualitatively equals that of the diquark model, the structure of the eigenstates is quite different; mainly because the wave functions contain significant contributions from higher-dimensional  $SU_F(3)$  representations as *e.g.* the  $\mathbf{27}$ -plet. We are thus led to the conclusion that a two-component mixing scenario can only be a first approximation to the description of pentaquark-like baryon resonances.

The present treatment also allows one to address the large- $N_C$  discussion that has recently emerged for the study of pentaquarks in chiral soliton models. Vibrational excitation energies are  $\mathcal{O}(N_C^0)$ , so are the  $SU_F(3)$  rotational excitation energies. While the leading order cancels for mass differences between baryons built from  $N_C$  quarks, it does not for exotics [25] that have  $N_C + 1$  quarks and one antiquark. This indicates that for the discussion of pentaquarks rotational and vibrational modes should be considered on equal footing and that mixing effects may become an issue. Since the transition operator between these modes originates from the flavor symmetry breaking part in the Lagrangian, the corresponding matrix element is potentially small. Nevertheless the approximate equality of the excitation energies may trigger sizable mixing effects. In the vibrational treatment, the electric monopole (scaling) and the magnetic dipole (rotations) channels contribute to the scattering amplitude in the partial waves in which the low-lying  $J^\pi = \frac{1}{2}^+$  resonances occur [36, 39]. In the present study we have considered exactly these two degrees of freedom. However, we did not treat them as RPA modes, *i.e.*  $\mathcal{O}(\hbar\omega)$  excitations, but went beyond such a quadratic approximation for the fluctuations off the soliton. In this treatment we have indeed seen that the leading excitation energies of these modes are almost identical. And, not surprisingly, the mixing is important for the excitations of octet and decuplet states. On the other hand the predicted masses for exotic states that do not have partners with equal quantum numbers in the octet or decuplet did not change significantly when

we included the monopole degree of freedom. We therefore conclude that the purely rotational treatment of such states reliably approximates their mass differences with respect to the nucleon.

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